130076

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FINAL REPORT

National Aeronautics and Space Administration

Grant NGR 22-007-056

THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF ANTENNAS AND WAVES IN PLASMA

(NASA-CR-130076) THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF ANTENNAS AND WAVES IN PLASMA Final Report, 1 Sep. 1965 - 31 Dec. 1970 (Harvard Univ.) 12 Jun. 1972 21 p CSCL 17B G3/07
N73-10184
Unclas
15978

For the period

September 1, 1965 through December 31, 1970

June 12, 1972

Amount of funds awarded: $222,681

NOV 1972
RECEIVED

Prepared for

Grants and Research Contracts Division
Office of Space Science and Applications
National Aeronautics and Space Administration
Washington, D.C. 20546

Theoretical and Experimental Investigations on Plasma-Coated Antennas - C. Y. Ting and B. Rama Rao.

The aim of this research project has been to investigate both theoretically and experimentally the near-field characteristics of an antenna when it is covered by a layer of ionized gas, i.e., a plasma sheath. As evidenced by re-entry vehicles, this plasma sheath can greatly affect radio communication. The theoretical study has been concerned with the determination of the current distribution, input admittance, and radiation pattern as functions of the electrical length of the antenna. Experimental work was directed toward measuring the current distribution and input admittance at frequencies below, near, and above the plasma frequency. Diagnostic measurements were also made to determine the properties of the plasma.

In the theoretical analysis, a numerical method was used to solve the problem of a finite cylindrical antenna in an infinite plasma column. To make the problem mathematically tractable, a highly idealized plasma model was assumed: collision losses and electron temperature effects were ignored, and the plasma column was assumed to be radially homogeneous.

Extensive experimental measurements were made to verify the theory. The plasma sheath surrounding the antenna was produced by means of a hot-cathode, d.c. discharge contained within a long coaxial glass tube. The inner glass tube served as a d.c. insulating sleeve around the antenna so that no d.c. current was drawn from the discharge by the antenna. The current distribution and input admittance of the antenna were measured by a small shielded loop probe which traveled along a narrow slot in the antenna. The electron density, electron temperature, and collision frequency were measured using Langmuir probes and the microwave-cavity perturbation technique.

Because of practical limitations in the design and construction of the experimental apparatus, it was not possible to generate a homogeneous, lossless, cold plasma column of infinite length to correspond to the simplified theoretical model. In practice, it was possible to produce a plasma column only a few wavelengths long. The electron-density profile across the plasma tube was diffusion controlled and far from uniform; the electron temperature was very high; and the losses due to collisions were considerable, especially at high pressures. It was also necessary to produce the plasma inside a glass container so that glass walls and an air gap were interposed between the antenna and the plasma sheath. Furthermore, measurements of the electron density by the Langmuir probe near the antenna show that the plasma in the annular region around the antenna does not have angular symmetry. None of these effects was taken into account in the theoretical calculations and, as a result, a meaningful quantitative comparison of the predicted theoretical results with the measured data was not possible.

The experimental results do show, however, a reasonable degree of qualitative agreement with the theoretical predictions. The general characteristics of a finite, cylindrical, plasma-coated antenna can be summarized as follows. It was found that when $0 < \varepsilon_r < 1$ ($\omega > \omega_p$), the plasma behaves essentially like a lossless medium and the presence of the plasma sheath tends to shorten the effective electrical length of the antenna. The current distribution is still approximately sinusoidal but with a longer wavelength. In the range $-1 < \varepsilon_r < 0$ ($\omega_p > \omega > \omega_p/\sqrt{2}$), the antenna characteristics undergo a drastic change. Due to the presence of the plasma sheath the current attenuates very rapidly along the length of the antenna. Within this same range, an antenna that is longer than a quarter-wavelength behaves very much like an infinitely long antenna with an input admittance that remains almost constant and independent of the antenna length. It was also noticed that the input resistance of a very short dipole shows a strong "resonance" phenomenon at a frequency below the plasma frequency.

Theoretical Studies on an Antenna Immersed in a Plasma - A. D. Wunsch.

One important present day use of antennas is in the field of ionospheric exploration. Man-made satellites are frequently equipped with dipole antennas which serve as probes to determine the constitutive parameters (electron density, magnetic field strength, temperature, etc.) of the earth's ionosphere from measurements of the antenna impedance. The purpose of this research effort has been to establish a theory to relate these impedance measurements to the ionospheric parameters.

One aspect of this investigation has treated the ionosphere as a cold, homogeneous, anisotropic medium. The plasma is characterized by a dielectric tensor, and the antenna, approximated as a strip having a prescribed current distribution, is assumed to be perpendicular to a uniform magnetic field which permeates the surrounding medium. An expression has been obtained for the radiation resistance of the antenna in terms of the parameters of the plasma (magnetic field strength and electron density) and numerical results have been presented for a broad range of operating frequencies. The limitations imposed by treating the antenna as an infinitesimal current element (Hertzian dipole) are discussed.

Another aspect of this research has treated the ionosphere as an isotropic, homogeneous, ionized gas of finite temperature. Unlike the previous problem, no assumption is made concerning the distribution of current along the antenna. The current distribution along the antenna has instead been determined and the input resistance and reactance of the antenna thereby obtained. Previous workers have made such warm plasma impedance calculations by using a distribution of current along the antenna which is identical to that which would exist if the plasma were cold. The current distributions obtained in this study indicate that, in general, this is an unjustified approximation. The far field radiation pattern of the antenna, based on the correct current distribution, has also been calculated.

Since a tubular antenna, driven on the outside and the inside, has been considered, it was decided to determine the relative sizes of the currents on the outside and inside of the tube. This is especially important since even a very thin tube containing warm plasma can support an internal propagating wave. Expressions have been derived from which the currents on the inside and outside surfaces of the tube can be determined once the total current is known. For the comparatively thin antennas considered in this analysis, it has been found that most of the total current lies on the outside surface of the tube. Thus, the results obtained for the tubular antenna would probably apply to a solid one as well.

Plasma Diagnostics Using a Modified Cavity Perturbation Technique - Y. S. Yeh and W. A. Saxton.

One of the common ways to determine the electron density and effective electron-neutral collision frequency of a gaseous plasma is to measure the perturbations of a microwave cavity due to the presence of the plasma within the cavity. In this technique the plasma, usually bounded by a glass discharge tube, is inserted into a cavity whose resonance frequency and bandwidth are known beforehand. When the plasma appears, the resonance frequency and bandwidth are perturbed and take on new values which depend on the plasma's geometrical configuration, its physical orientation within the cavity, the shape and dimensions of the cavity, and the distribution of the r.f.

fields within it. Based on the assumption that the plasma is a well-described dielectric with a relative dielectric constant and loss tangent dependent on the plasma and collision frequencies, the differences between the resonant frequencies and bandwidths, with and without the plasma present, are used to calculate the electron density and collision frequencies.

Since many laboratory discharge tubes are cylindrical in shape, it is convenient to choose a cylindrical cavity geometry. If the diameter of the discharge tube is much smaller than the diameter of the cavity, and if the field fringing at the holes on the faces of the cavity through which the tube is inserted is negligible, it is usually possible to get a relatively good measure of the plasma's equivalent dielectric constant and loss tangent. There are, however, situations in which the discharge tube has a large diameter compared to practical cavity dimensions, protrudes from holes in the cavity where sizable fringing fields are produced, and is not axially uniform as in the usual discharge. In cases such as these, if one were to use a cylindrical cavity it would have to be unusually large to apply the usual cavity perturbation theory, with resonance frequencies which would probably be too low for the range of plasma frequencies encountered. Furthermore, axial resolution of the plasma properties would be almost impossible since so much of the discharge would be inside the cavity.

One solution to the problem of diagnosing a discharge with these characteristics is to construct a very long rectangular cavity whose cross section has a relatively small dimension parallel to the axis of the discharge tube. As long as the cavity is narrow compared to the length of the discharge it will serve to resolve axial variations. The other dimensions can be adjusted according to the desired range of resonance frequencies and bandwidths. In most cases, however, it will be impossible to avoid a situation in which the discharge tube diameter is comparable to, and not much less than, at least one of these dimensions and/or the cavity width in which case fringing at the holes becomes more complex.

An investigation was made to analyze the cavity perturbation method for a thin rectangular cavity with a large plasma tube protruding from both of its sides. A modified theory has been developed for a general plasma column in which the gross disturbances of the fields at the holes are taken into account as well as the fact that certain cavity dimensions are comparable to and even smaller than the plasma column diameter. A rectangular cavity having smaller overall dimensions than a cylindrical cavity with the same range of resonance frequency and bandwidth was constructed. Experimental data were taken on dielectric constant and loss tangent for several sample liquid and solid dielectrics. This data corroborated the theoretical predictions based on the modified theory and indicated the applicability of this general approach to actual plasmas. Plasma measurements were then performed which yielded data on electron densities that agreed with separate measurements using Langmuir probes.

Experimental and Theoretical Study of the Diffraction of Electromagnetic Waves by Moving Striations in a Plasma Column - B. Rama Rao and W. A. Saxton.

Moving striations are periodic, macroscopic waves of ionization moving within the positive column of the glow discharge of an inert gas. Experimental and theoretical investigations have been made to study the diffraction of microwaves by large amplitude, regular striations inside a plasma column. The phenomenon observed here is similar to the Bragg scattering of light by means of elastic waves in liquids and solids. The spectrum of the diffracted electromagnetic wave consists of a doublet split symmetrically around the incident frequency by an amount equal to the fundamental striation frequency and its higher harmonics. Because of the non-linear nature of the large amplitude, natural striations, 'Bragg doublets' due to several harmonics of the striation frequency have been observed. A knowledge of the scattering angle enables one to determine the wavelength and velocity of the striations. The results for the striation velocity determined from the diffraction measurements are in good agreement with independent measurements made with a photomultiplier tube. Because of the saw-tooth nature of the electron density profile along the axis of the tube, the diffraction pattern is asymmetric. By measuring the relative intensities of the two doublets, the shape of the electron density profile can be determined.

The diffraction measurements were made at K-band microwave frequencies of 25 Gcs. (λ = 1.2 cms), with the electric vector of the incident electromagnetic wave polarized perpendicular to the axis of the plasma column. The diffracted signal was detected by using a superheterodyne system in conjunction with an audio frequency wave analyzer. The plasma column was a hot-cathode Argon discharge, 50 cms in length and 3.3 cms in diameter. The neutral gas pressure and discharge current were varied until the discharge operated in a stable, well-defined striation mode. The striations were then controlled and stabilized by using various experimental techniques. Diffraction patterns of up to the eighth harmonic of the fundamental striation have been measured. It has been observed that the striations are backward waves, with phase velocities directed toward the cathode. Typical values of the striation velocity measured in this experiment varied from 200 meters/second at a pressure of 1.9 mms Helium to about 480 meters/second at 190 μ Helium.

For striations propagating within a finite plasma column the diffraction pattern has been calculated theoretically by using a Born approximation procedure. The intensity of the diffracted electromagnetic wave depends on the striation wavelength, the polarization of the electromagnetic signal and the radial electron density profile within the plasma column. This investigation offers a convenient microwave technique for detecting and analyzing moving striations.

Parametric Interactions in a Dielectric Medium Space-Time Modulated by Two Pump Waves - B. Rama Rao.

In recent years several authors have considered the propagation characteristics of electromagnetic waves traveling in media with properties varying periodically in time and space. In all of these cases, it is tacitly assumed that the perturbing wave modulating the medium is a weak, progressive, sinusoidal disturbance. However, there are many applications in ultrasonics, quantum electronics, and in plasma and ionospheric physics where the modulating pump wave has a large amplitude and contains several harmonics of the fundamental pumping frequency. Alternatively, the dielectric medium can be subjected to "parallel pumping" by two pump waves of incommensurate frequencies. Electromagnetic wave propagation in dielectric media of this type has been analyzed. To keep the analysis simple, only the first two harmonics of the pump wave have been considered.

It has been assumed that the high intensity pump wave (either acoustic or microwave phonon wave) traveling along the z direction in an infinite dielectric medium contains two pump harmonics and causes the dielectric constant of the medium to vary in the following manner:

$$\varepsilon(z,t) = \varepsilon_0 + \varepsilon_1 \cos(\omega_V t - k_V z) + \varepsilon_2 \cos(2\omega_V t - 2k_V z)$$

where $\varepsilon_2 << \varepsilon_1 << \varepsilon_0$. ω_V and k_V are, respectively, the angular frequency and the wavenumber of the fundamental component of the pump wave. A dispersion relation in the form of continued fractions involving all space-time harmonics has been obtained for electromagnetic wave propagation in an infinite dielectric medium. The convergence criterion has been established from Poincarē's theorem and a 'sonic' region has been defined. The physical parametric processes involved are explained in terms of equivalent non-linear polarization sources postulated by Bloembergen [Non-Linear Optics, New York, W. A. Benjamin & Co., 1965]. The second harmonic modulating coefficient ϵ_2 causes strong parametric coupling between the first-order stokes and antistokes waves and also between the fundamental and second-order antistokes waves. Convective type instabilities that occur when the phase velocity of the pump wave exceeds the phase velocity of the signal wave are also considered. When the two pump waves are excited independently by two different generators, the relative phase difference between the waves affects the parametric coupling in the stop-band region. The theory which has been developed is used to explain some experimental observations on the non-linear scattering of microwaves by large amplitude density-fluctuations in a plasma.

Plasma Profile Investigations Using a Multi-Mode Cavity Perturbation Technique - B. Rama Rao and L. D. Scott.

The cavity perturbation method has been widely used as a diagnostic technique for measuring electron densities of plasmas. The accuracy of this method depends largely on the assumed electron density profile distributions used in the calculations. The profile distribution commonly used is the Schottky diffusion profile of the type $J_0(2.404r/r_w)$ where r_w is the radius of the plasma column. This type of profile, however, is valid only at high gas pressures (ambipolar diffusion limited case) when the mean free path is small compared to the radius r_w of the plasma column. Substantial errors can result if the same density profile is used for measurements at low pressures or when the mean free path is comparable to r_w.

The purpose of this study has been to use a multi-mode cavity perturbation scheme so that the plasma diagnostics could be extended over a much wider range of pressures. Conversely, this method has also been employed to study the plasma-sheath problem and to determine the variation in electron density profiles as a function of pressure and electron temperature. The method consists in expressing the profile function as a power series of the radius and evaluating the coefficients by measuring the shifts in the resonant frequencies of the appropriately chosen cavity modes. For example, in the high pressure regime the electron density profile, after taking into account both ambipolar diffusion and volume recombination, is of the form $n(r) = n_0 - n_2(r/r_w) + n_4(r/r_w)^2$. The profile coefficients n_0, n_2 and n_4 have been determined by measuring the frequency shifts of three different cavity modes, with different electric field distributions in the radial directions. The modes chosen in the experiment were the TM_{020}, TM_{120} and TM_{210}; additional checks were made with the TM_{012} and TE_{212} modes. Experimental measurements indicate that at high pressures the electronic density profile is of the Schottky diffusion type; as the pressure becomes lower, the gas-kinetic collisional frequency is too low to set up diffusion and the electron density profile shows some marked deviations. For very low pressures, measurements of the second radial moment of the electron density distribution made on a hot-cathode discharge tube using helium compared favorably with previous theoretical results. The electron temperature and electron density at the axis of the tube were measured by using Langmuir probes.

A Short Cylindrical Antenna as a Diagnostic Probe for Measuring Collision Frequencies in a Collision-Dominated, Non-Maxwellian Plasma - L. D. Scott and B. Rama Rao.

Experimental and theoretical investigations have been made to determine the effects of interparticle collisions on the antiresonant impedance characteristics of an electrically short antenna in the vicinity of the plasma frequency. The use of this antenna as a diagnostic probe for measuring effective collision frequency and electron density has been carefully explored using the theories proposed by King, Harrison and Denton [J. Res. NBS, Vol. 65D, No. 4, pp. 371-384, 1961] and by Balmain [Radio Science, Vol. 69D, No. 4, pp. 559-566, 1965]. The results of this technique are in good agreement with other well accepted diagnostic methods, when suitable corrections are introduced for antenna-transmission line junction effects. This investigation substantiates the recent observations made by Larson [Radiation Lab. Tech. Rept. 7000-25-T, Univ. of Michigan, Ann Arbor, pp. 117-122, September 1966] which indicate that near the plasma frequency the influence of collision losses on the antenna resistance are similar to that obtained from including electroacoustic wave effects. Measurements were made at frequencies from 300 to 600 MHz in a hot-cathode Helium discharge at neutral gas pressures ranging from 35 to 300 μ of Helium. The antenna was a cylindrical copper rod, 3.5 cm in length and 4 mm in diameter.

For estimating electroacoustic mode effects the electron temperature was measured using a Langmuir probe. In a collision-dominated plasma like the one used in this investigation the electron temperature effects calculated from Balmain's theory were found to be quite small. It was also determined experimentally by varying the d.c. bias applied to the antenna that near the plasma frequency the ion sheath contributes negligibly to the antenna impedance when the electron density and collision frequency are large. Hence, under these experimental conditions, it is justifiable to use a simple, cold plasma model in the analysis of antenna characteristics.

Theoretical values of the electron-neutral and electron-ion collision frequencies were calculated by using the collision cross-sections for Helium as measured by Golden and Bandel [Physical Review, Vol. 138, No. 1A, pp. A14-A21, 1965]. The effective collision frequency was then related to the complex dissipation factor used in Langevin's equation for defining the plasma conductivity as suggested by Molmud [Physical Review, Vol. 114, No. 1, pp. 29-32, 1959]. The measured values of collision frequencies agree within a factor of less than two in this investigation. This degree of agreement is considered quite good, if one considers possible impurities in the gas, errors in determining electron temperature and neutral gas pressure, and inhomogeneities in the plasma column.

Theoretical and Experimental Studies on an Antenna in a Magnetoplasma - Bharathi Bhat and B. Rama Rao.

A study to determine theoretically and measure experimentally the electrical properties of an antenna in a magnetoplasma has been completed. Practical applications of this problem include the use of dipole antennas mounted on satellites or rockets for communication through the earth's ionosphere and magnetosphere as well as for purposes of probing the properties of an ionized medium; the interpretation of cosmic noise intensity measurements and radio astronomical measurements from space vehicles; and in laboratory plasma diagnostics. An understanding of the precise relationship between the electrical properties of the antenna and those of the medium is essential.

The various assumptions and approximations made in previous theoretical analyses preclude the consideration of antenna behavior over a wide range of magnetoplasma parameters especially near the resonance and cut-off conditions. The behavior of the antenna impedance near these critical frequencies is conspicuously different from that of an antenna in free space. In the integral formulation for the current on a finite tubular antenna in isotropic media the propagation constant of the medium is the same as that for the current wave on the antenna. For a magnetoplasma the situation is not as simple since there is more than one characteristic wave and each is characterized by a different propagation constant. Furthermore, an antenna in air can be assumed sufficiently thin to guarantee that all the waveguide modes are below cut-off and that therefore, at distances from the driving point, the total current is equal to the outside current. A similar argument is not applicable for the situation in which the antenna is immersed in a magnetoplasma. In each of the resonant regions of a collisionless plasma, the refractive index of one of the characteristic waves goes to infinity at certain angles to the direction of the d.c. magnetic field. Since the corresponding characteristic wavelength approaches zero, the antenna appears electrically thick. Under these conditions the tube can support a large number of waveguide modes regardless of how thin the antenna may actually be physically. The quasi-static approximation becomes invalid in the resonant regions and the actual current distribution on the antenna differs significantly from the simple triangular distribution assumed in several analyses. The assumption of a sinusoidal distribution of current presupposes a knowledge of the propagation constant for the current along the antenna and the latter question has not yet been completely resolved. The uniaxial approximation requires that the off-diagonal terms of the permittivity tensor characterizing the magnetoplasma be small in comparison with the leading diagonal term which is true only when the applied magnetic field is quite large. When the plasma parameters approach cut-off, however, the value of the off-diagonal term of the permittivity tensor becomes almost equal in magnitude to the leading diagonal term and the refractive index of one of the characteristic waves becomes zero. It appears therefore that the antenna current distribution can vary considerably over the resonance and cut-off regions.

The theoretical part of this research has been concerned with examining the current distribution in an infinitely long tubular antenna immersed in a cold collisional magnetoplasma with d.c. magnetic field oriented parallel to the antenna. Based upon a boundary-value formulation, an integral equation has been derived for the total axial current on such an antenna. It has been shown that even the slightest collisional loss eliminates the infinities in the refractive index of the characteristic waves that are possible inside the medium. This supports the hypothesis that waveguide modes are not present within the tube and, therefore, that the antenna is electrically thin even in the resonant regions. The expression for the current has been solved numerically for a wide range of plasma parameters. In particular, results are presented for the current distributions in the resonant and non-resonant regions for varying levels of magnetic field intensity and for different values of the collision frequency.

It was found that when collisional losses are small ($\nu/\omega_p < 0.1$) the antenna current over the regions where $|\mathrm{Re}\,\sqrt{K_\perp}| >> |\mathrm{Im}\,\sqrt{K_\perp}|$ (i.e., $K_\perp > 0$ for $\nu/\omega_p = 0$) is essentially sinusoidal with a phase constant very close to $\mathrm{Re}\,k_0\sqrt{K_\perp}$ and that its amplitude factor is proportional to $K_\perp$ (at least over a length $k_0 z < \pi$). The trend in the current close to the driving point agrees with the experimental findings on the input impedance of a finite antenna which show that the input impedance has a maximum near the upper hybrid resonance and a minimum near the cyclotron resonance. Over the regions where $|\mathrm{Re}\,\sqrt{K_\perp}| << |\mathrm{Im}\,\sqrt{K_\perp}|$ (i.e., $K_\perp < 0$ for $\nu/\omega_p = 0$) the current away from the driving point decays exponentially according to $e^{-\alpha z}$ where α is approximately given by $|\mathrm{Im}\,k_0\sqrt{K_\perp}|$. When the collisional losses are high ($\nu/\omega_p \gtrsim 0.1$), neither the traveling-wave nature characteristic of regions where $K_\perp > 0$ nor the fast exponential decay characteristic of regions where $K_\perp < 0$ is predominant; rather the currents approach similar distributions.

An experimental investigation has been made of a cylindrical antenna, whose length is short compared with the free space wavelength, immersed in a collisional magnetoplasma with d.c. magnetic field oriented parallel to the antenna axis. The input impedance of the antenna was studied at and in the vicinity of the various transition regions of a magnetoplasma resonance and cut-off. The antenna operating frequency was varied (typically from 250 to 600 MHz) while the plasma, cyclotron, and collision frequencies were maintained at constant values by fixing principally the discharge current, solenoid current, and neutral gas pressure, respectively. Measurements were made at various magnetic field strengths. The effect of collisions on antenna impedance was studied by repeating the measurements at several neutral gas pressures. Diagnostic measurements were made using a Langmuir probe to determine the electron density and electron temperature and using a Hall-effect probe to determine the strength of the applied magnetic field so that the prevailing experimental conditions could be well defined.

The following general trends in antenna impedance behavior were observed experimentally: 1) A dominant resistance peak occurs near the upper hybrid resonance. 2) A dominant conductance peak occurs near the cyclotron resonance. 3) A small resistance increase occurs near the plasma cut-off. 4) The reactance is negative for frequencies below f_c (electron cyclotron frequency), positive between f_c and f_p (electron plasma frequency), and negative above f_p (for $f_c < f_p$). 5) Increasing collisions causes a considerable shift of the resistance maxima and reactance zeros from their corresponding locations for no collisions. It is interesting to note that exceptions to the foregoing general trends do arise under certain conditions. For example, if $f_c < 0.2f_p$, the upper hybrid frequency and plasma frequency are so close to one another that their resistance maxima merge to yield only a single maximum. If $f_c \sim f_p$, the two opposing effects on resistance are superimposed, thus somewhat obliterating the plasma cut-off but not affecting the conductance peak near the cyclotron resonance which is still observed. Also the reactance may not necessarily change sign. If f_c is sufficiently close to f_p and the magnetic field strength or the collision frequency is large, the reactance remains capacitive throughout the entire frequency range.

If the strength of the magnetic field is measured independently, the resistance maximum which is always observed in the vicinity of the upper hybrid resonance offers a reliable method of determining the plasma frequency and the effective collision frequency by matching the theoretical and experimental resistance maxima. If the magnetic field is also an unknown factor, then the same iteration scheme may be used alternately to match the conductance peak in the vicinity of the cyclotron resonance and the resistance peak in the vicinity of the upper hybrid resonance to determine the plasma frequency, cyclotron frequency, and effective collision frequency.

Linear Antennas in Lossy Media - L. D. Scott

A practical experimental environment has been designed using a natural body of fresh water together with a thin polyethylene-walled tank which effectively approximates an infinite, homogeneous, isotropic, dissipative medium for studying the electrical properties of relatively thin ($h/a \gtrsim 20$ to

200) linear antennas immersed in such a medium. The ratio α/β of the medium may range from 0.016 to very near 1. An equipment float was designed to accommodate at least two researchers together with the necessary electronics. This float serves as the support structure for the ground plane which images the various length monopoles used in the study. The float also supports the polyethylene-walled tank beneath it.

A special technique was developed to measure more accurately the effective conductivity and effective dielectric constant of the medium in which the monopoles are immersed. This involves a low frequency determination of an approximate value for σ together with a high frequency determination of β from which the values of σ and ε are found. The accuracy is improved by making the determining measurements under more ideal conditions for each component of data.

A simple and efficient, yet precise, admittance-measuring technique was employed which involved the probing of fields on a precision section of transmission line at a precisely known distance from the driving point of the antenna. This system was self-calibrating after its initial laboratory calibration. It presented a continuous direct reading of the admittance magnitude and phase at the probing point. Over a six month period of operation in the laboratory and field, all calibration checks indicated the accuracy of measured $|Y|$ was ±2% with a phase accuracy of ±1.5°. The admittances of antennas with βh ranging from approximately 0.3 to 3.3 in media with α/β = 0.016 to 0.97 were measured and tabulated.

Current and charge measuring apparatus was designed to position precisely probes of very small electrical dimensions along the surface of the monopoles being investigated. The response errors of the magnetic probe were investigated to determine the range of validity of its assumed current response with respect to operation near the end and the driving point of the antenna. A redundancy of data is available for the current distribution since the charge distribution for a particular antenna may be integrated to get another independent determination of the current distribution. Reproducibility of measured results was found to be ±4% in the magnitude with a 2° difference in phase for a typical case. The current and charge distributions for antennas with βh ranging from approximately 0.3 to 3.3 in media with α/β = 0.016 to 0.97 were measured and tabulated.

Junction effects were investigated to determine the source of some probing errors and to find out if the abrupt air-water discontinuity at the antenna driving point caused an end correction in addition to that required with a continuous dielectric in the junction region. It was determined that it was not necessary to include any additional end corrections for the electrical dimensions of the apparatus used in this study. The effects of various b/a ratios in the coaxial line used to drive the antenna were investigated. The results were that for α/β near 0, only the susceptance is significantly affected by changing b/a, but that at α/β near 1 the conductance is also significantly affected by changing b/a. Thus, in general, G and B are dependent on b/a for all α/β.

A new polynomial-exponential-product (PEP) theory has been developed to solve Hallén's integral equation for the current distribution on a perfectly conducting, symmetrical, center-driven dipole immersed in an infinite, homogeneous, isotropic, dissipative medium. It is an approximate solution using expansion functions for the current distribution that have been optimized by statistically analyzing measured antenna current distributions. In addition, the expansion functions are constrained to be non-singular elementary functions whose coefficients have obvious physical interpretation. The usual singularities that occur in the exact solution of the integral equation due to the delta-function generator are eliminated by purposely choosing non-singular expansion functions. Polynomials in z are initially selected as the expansion functions because they possess the above properties and are able to reproduce the measured current distributions for antennas in non-dissipative media. Then, an exponential term multiplying the polynomial is used to produce the more rapid current decay observed for antennas in dissipative media. The argument of the exponential is selected *a priori* to be $-\alpha|z|$ where α is the attenuation constant for plane-wave propagation in the medium. An analysis of antennas in sea water (highly dissipative medium) has shown that this selection is quite appropriate. The solution is general in that it is applicable to dipoles with $\beta h \leq \pi$ in media with $\alpha/\beta \leq 5$. Thus, it covers a range of antenna lengths of considerable interest in isotropic, homogeneous media characterized by an effective conductivity and dielectric constant (e.g., free space, earth, sea water, cool plasmas.)

Measured admittances for antennas with $\beta h \approx 0.3$ to 4.7 in media for which $\alpha/\beta = 0$ to 0.97 are compared with the theoretical results of the PEP theory, Wu's long antenna theory [J. Math. Phys., Vol. 2, pp. 550-574, July-

August 1961] and King's three-term theory [J. Appl. Phys., Vol. 40, pp. 5049-5065, December 1969]. Measured current and charge distributions for selected antennas immersed in dissipative media are compared with the PEP theory and King's three-term theory. Wu's theoretical results were found to be in excellent agreement with the measured conductances and within a constant of the measured susceptances. King's three-term theory was found to be in good agreement with the measured results for current distributions with its charge distributions somewhat less accurate. The PEP theory appears to have the best overall agreement (i.e., admittance, current and charge) to the measured results presented in this study. All three solutions have to some extent the errors inherent in the integral equation formulation with the delta-function source. Each solution through its approximations tends to subtract out the delta-function response.

The PEP method utilizes measured data of at least two judiciously selected antennas to optimize the choice of the current expansion function and the optimum collocation points in the approximate solution of the integral equation. With this limited initial data the PEP method is able to predict the electrical properties of a whole class of antennas. It is evident from the comparisons with experiment and the Wu and King theories that this method can yield very good results. Disadvantages of this method are 1) that it requires a certain initial amount of measured data in order to be implemented; and 2) that, unlike the theories of Wu and King which have been derived from mathematical considerations with each approximation rigorously justified, the PEP method involves an expansion function in an approximate solution about which no *a priori* accuracy can be assigned. An advantage of the general method used in the PEP theory is that it can be applied to situations where analytical techniques are unable to yield results.

Investigations have been made to determine the effects of interparticle collisions on the antiresonant impedance characteristics of an electrically short, cylindrical antenna in the vicinity of the plasma frequency of an isotropic, non-Maxwellian plasma. The dependence of the electron-neutral collision frequency on the electron energy has been taken into account. The experimental results are compared with the theories proposed by King *et al.* [J. Res. NBS, Vol. 65D, No. 4, pp. 371-384, 1961] and Balmain [Radio Science, Vol. 69D, No. 4, pp. 559-566, 1965]. The use of this antenna as a diagnostic probe for measuring electron-neutral collision frequency and electron density has been investigated. Electron-neutral collision frequencies for helium measured by this technique are in good agreement with theoretical results calculated from the collision cross-section data of Golden and Bandel [Phys. Rev., Vol. 138, pp. A14-A21, April 1965]. These investigations have shown that 1) an electrically short cylindrical antenna can be used for diagnosing the electron density and collision frequency of the plasma; 2) near the plasma frequency the influence of collisional losses on the antenna resistance is similar to that obtained from including electroacoustic wave effects; 3) the resistance contribution due to the electroacoustic mode is negligible when the collision frequency is high, so that the "cold" plasma theory is applicable for interpreting experimental data; 4) the velocity dependence of the collision frequency can be accounted for satisfactorily by introducing a complex effective collision frequency; and 5) the effect of the ion-sheath on the antenna impedance near ω_p is small when the electron density and collision frequency are large.

Antennas in Warm Isotropic Plasmas - D. H. Preis and M. V. Ward.

Because the circuit properties of the bare antenna immersed in a warm, isotropic plasma are not completely understood, credible r.f. plasma diagnostics using antennas are not really possible. Research was initiated in this general area in order to ascertain the range of validity for different plasma models and antenna theories. Although there has been considerable theoretical speculation of this subject, no comprehensive as well as closely correlated theoretical and experimental investigation had ever been presented. Following an extensive review of the literature, it was concluded that previous theoretical research in this area incorporated questionable assumptions, most notably the validity of the fluid-dynamical equations, the treatment of collisional phenomena as well as certain aspects of the boundary conditions at the surface of the antenna. Under NASA Grant NGR 22-007-056 theoretical research was initiated from the kinetic theory point of view, boundary-value problems were investigated, and the design and fabrication of experimental apparatus capable of measuring distributions of current and charge on bare antennas in warm, isotropic, laboratory plasmas was begun. Since the termination of the NASA Grant theoretical work has been continued and preliminary experimental data have been taken with the support of NSF Grant GK-24105.

LIST OF SCIENTIFIC REPORTS ISSUED UNDER NASA GRANT NGR 22-007-056

No. 1. C. Y. Ting, B. Rama Rao, and W. A. Saxton, "Theoretical and Experimental Study of a Finite Cylindrical Antenna in a Plasma Column," July 1967.

No. 2. B. Bhat and B. Rama Rao, "Experimental Investigations on the Impedance Behavior of a Short, Cylindrical Antenna in a Lossy Magnetoplasma," February 1969.

No. 3. L. D. Scott and B. Rama Rao, "A Short, Cylindrical Antenna as a Diagnostic Probe for Measuring Collision Frequencies in a Collision-Dominated, Non-Maxwellian Plasma," March 1969.

No. 4. B. Rama Rao, "Parametric Interactions in a Dielectric Medium With Second-Harmonic Space-Time Pump Modulation," August 1969.

No. 5. Y. S. Yeh and W. A. Saxton, "Modified Theory for Cavity Perturbation Measurement of Plasma Parameters," October 1969.

No. 6. L. D. Scott, "Apparatus for Studying the Properties of Antennas in an Effectively Infinite Dissipative Medium," December 1969.

No. 7. L. D. Scott, "Polynomial-Exponential-Product Theory for Antennas in Homogeneous Isotropic Media," December 1969.

No. 8. L. D. Scott, "Comparisons of Measured Admittances, Current Distributions, and Charge Distributions with the PEP Theory, King's Three-Term Theory, and Wu's Long Antenna Theory," December 1969.

LIST OF PUBLICATIONS REPORTING WORK SUPPORTED BY NASA GRANT NGR 22-007-056

A. D. Wunsch, "Radiation from a Strip of Electric Current in a Magnetoionic Medium," Canadian Journal of Physics, Vol. 45, pp. 1675-1691, 1967.

A. D. Wunsch, "Current Distribution on a Dipole Antenna in a Warm Plasma," Electronics Letters, Vol. 3, pp. 320-321, July 1967.

C. Y. Ting, B. Rama Rao, and W. A. Saxton, "Theoretical and Experimental Study of a Finite Cylindrical Antenna in a Plasma Column," IEEE Trans. Antennas Propagat., Vol. AP-16, pp. 246-255, March 1968.

D. Lamensdorf and C. Y. Ting, "An Experimental and Theoretical Study of the Monopole Embedded in a Cylinder of Anisotropic Dielectric," IEEE Trans. Antennas Propagat., Vol. AP-16, pp. 342-349, May 1968. [Also supported by Air Force Cambridge Research Laboratories Contract AF19(628)-2406].

A. D. Wunsch, "The Finite Tubular Antenna in a Warm Plasma," Radio Science, Vol 3 (New Series), pp. 901-919, September 1968.

B. Rama Rao, "Electromagnetic Wave Propagation in a Dielectric Medium Space-Time Modulated by a Non-Linear Pump Wave," Proc. of the IEEE, Vol. 56, pp. 1630-1631, September 1968.

L. D. Scott and B. Rama Rao, "A Short Cylindrical Antenna as a Diagnostic Probe for Measuring Collision Frequencies in a Collision-Dominated, Non-Maxwellian Plasma," IEEE Trans. Antennas Propagat., Vol. AP-17, pp. 777-786, November 1969. [Recipient of the Best Paper Award for 1969 from the IEEE Transactions on Antennas and Propagation.]

B. Rama Rao, "Parametric Interactions in a Dielectric Medium with Second-Harmonic Space-Time Pump Modulation," Proc. of the IEEE, Vol. 57, pp. 2173-2174, December 1969.

R. W. P. King and L. D. Scott, "The Cylindrical Antenna as a Probe for Studying the Electrical Properties of Media," IEEE Trans. Antennas Propagat., Vol. AP-19, pp. 406-416, May 1971. [Also supported by NSF Grant GK-3800 and Joint Services Electronics Program Contract N00014-67-A-0298-0005.]

B. Bhat and B. Rama Rao, "Experimental Investigations on the Impedance Behavior of a Cylindrical Antenna in a Collisional Magnetoplasma," submitted for publication in the IEEE Transactions on Antennas and Propagation.

LIST OF PAPERS PRESENTED AT SCIENTIFIC MEETINGS

1967 Spring URSI Meeting held in Ottawa, Ontario in May 1967:

B. Rama Rao and L. D. Scott, "Plasma Profile Investigations Using a Multi-Mode Cavity Perturbation Technique."

C. Y. Ting and B. Rama Rao, "Theoretical and Experimental Study of a Finite Cylindrical Antenna in a Plasma Column."

A. D. Wunsch, "The Current Distribution for a Dipole Antenna in a Warm Plasma."

1968 Spring URSI Meeting held in Washington, D.C. in April 1968:

B. Rama Rao and W. A. Saxton, "Diffraction of Electromagnetic Waves by Moving Striations in a Plasma Column - Experimental and Theoretical Studies."

1968 Fall URSI Meeting held in Boston in September 1968:

B. Rama Rao, "Electromagnetic Wave Propagation in a Dielectric Medium Space-Time Modulated by a Non-Linear Pump Wave."

1968 Fall International G-AP Symposium held in Boston in September 1968:

L. D. Scott and B. Rama Rao, "A Short Cylindrical Antenna as a Diagnostic Probe for Measuring Collision Frequencies in a Collision-Dominated, Non-Maxwellian Plasma."

1969 Spring URSI Meeting held in Washington, D.C. in April 1969:

B. Bhat and B. Rama Rao, "Experimental Investigations on the Impedance Behavior of a Short Cylindrical Antenna in a Magnetoplasma."

Fourth NASA Plasma Sheath Symposium held at Langley Research Center, Hampton, Virginia in October 1970:

L. D. Scott, B. Bhat and B. Rama Rao, "Investigations on a Cylindrical Antenna as a Diagnostic Probe for Isotropic and Magnetized Plasmas."

Equipment Inventory Report

Our records of equipment purchased under Grant NGR 22-007-056 show that only one item was purchased which had a cost of $1000 or more. This was a PRD Universal Klystron Power Supply 815-S1, purchased in October 1966 for $1050. To the best of our knowledge this piece of equipment is probably the PRD Universal Klystron Power Supply 815 now located in the Student Laboratory in the basement of Cruft Laboratory.

Report of Inventions

As Project Director under NASA Grant NGR 22-007-056 between President and Fellows of Harvard College and the National Aeronautics and Space Administration, I have examined and investigated the work performed under said Grant including progress and work reports for the period September 1, 1965 through December 31, 1970 and, by the authority of President and Fellows of Harvard College, hereby certify that to the best of my and their knowledge and belief, there were no Subject Inventions under said Grant during the aforesaid period which reasonably appear to be patentable.

Ronold W. P. King

Ronold W. P. King, Director
NASA Grant NGR 22-007-056